PSO 2 — Week 8: Problems

CS381, Fall 2026

Problem 1. Convex polygons

For points a, b, c in the plane, let

$$\text{orient}(a, b, c) \;=\; (b_x - a_x)(c_y - a_y) - (b_y - a_y)(c_x - a_x),$$

which is positive when a, b, c make a left (counterclockwise) turn, negative for a right turn, and zero when the three points are collinear. This is the test Graham's scan uses.

(a) You are given a convex polygon as its n vertices $v_0, v_1, \ldots, v_{n-1}$ in counterclockwise order, no three of them collinear. Given a query point q, decide in $O(\log n)$ time whether q lies inside the polygon or on its boundary. Say exactly where your algorithm uses convexity, and which step would go wrong on a polygon that is not convex.

(b) *(harder)* Show that any algorithm computing the convex hull of n points, with the hull's vertices listed in counterclockwise order, can be used to sort n distinct numbers $x_1, \ldots, x_n$ with only $O(n)$ extra work. (Hint: consider the points (x_i, x_i^2).) What does this say about the running time of convex hull algorithms? Which feature of the hull's output does your reduction rely on?